ORE in Pricing of Bermudan Swaptions:
Client Experience from Model Validation

Dr. Dmitry Zaykovskiy
Valuation Financial Instruments
Deutsche Pfandbriefbank AG

ORE User Meeting – Frankfurt, 23/11/2018
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Main points

- pbb and Quaternion joint venture on pricing of Bermudan swaptions
- Hull-White 1F (LGM)
- In-depth performance analysis
Agenda

1. Background
2. Hull-White Model
3. Mean Reversion Parameter
4. Final remarks
Background

Bermudan swaption project

- Swap until final maturity
- Option to cancel swap => Bermudan Swaption
- Price is sensitive to the intertemporal correlation

- Reach portfolio of Bermudan callable swaps
- Daily prices from major investment banks in collateral management
- Analysis of model and market prices is possible
One Factor Hull-White Model

Definition

• Short rate process SDE

\[ dr(t) = (\theta(t) - \alpha r(t)) \, dt + \sigma(t) \, dW(t) \]

\( \theta(t) \) - a function may be calculated from the discount factors

\( W(t) \) - standard Brownian motion

\( \sigma(t) \) - piecewise constant model volatility (vector)

\( \alpha \) - mean reversion parameter (scalar)

• Short rate \( r(t) \) is normally distributed

\[ r(t) \sim \mathcal{N}\left( e^{-\alpha t} r(0) + \frac{\theta}{\alpha} \left(1 - e^{-\alpha t}\right), \frac{\sigma^2}{2\alpha} \left(1 - e^{-2\alpha t}\right) \right) \]
One Factor Hull-White Model
Calibration of model parameters

\[ dr(t) = (\theta(t) - ar(t)) dt + \sigma(t) dW(t) \]

- Model volatility \( \sigma(t) \)
  - Calibrated on co-terminal European swaptions for given strikes
  - Has as many “steps” as calibrating swaptions
  - Iteratively stripped to match prices of all swaptions

- Mean Reversion \( \alpha \)
  - Controls intertemporal correlation
  - “Historically estimated”
  - “Implied to produce certain volatility shape”
  - “Somehow set”
  - Ultimately freely selectable, or ?
Mean Reversion Parameter
Example 1: Expectation

\[ r(t) \sim \mathcal{N} \left( e^{-\alpha t}r(0) + \frac{\theta}{\alpha} \left( 1 - e^{-\alpha t} \right), \frac{\sigma^2}{2\alpha} \left( 1 - e^{-2\alpha t} \right) \right) \]

\[ \text{corr}(r(T_1), r(T_2)) = \sqrt{\frac{e^{2\alpha T_2} - 1}{e^{2\alpha T_1} - 1}} \]

\[ \sigma = 0.65\% \]
\[ \theta = 0.03\% \]
\[ r(0) = -0.40\% \]
Mean Reversion Parameter

Example 2: Variance

\[ r(t) \sim \mathcal{N} \left( e^{-\alpha t} r(0) + \frac{\theta}{\alpha} (1 - e^{-\alpha t}) , \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha t}) \right) \]

\[ \text{corr}(r(T_1), r(T_2)) = \sqrt{\frac{e^{2\alpha T_2} - 1}{e^{2\alpha T_1} - 1}} \]

\[ \sigma = 0.65\% \]
\[ \theta = 0.03\% \]
\[ r(0) = -0.40\% \]
Mean Reversion Parameter

Example 3: Correlation

\[ r(t) \sim \mathcal{N} \left( e^{-\alpha t} r(0) + \frac{\theta}{\alpha} (1 - e^{-\alpha t}), \frac{\sigma^2}{2\alpha} (1 - e^{-2\alpha t}) \right) \]

\[ \text{corr}(r(T_1), r(T_2)) = \sqrt{\frac{e^{2\alpha T_2} - 1}{e^{2\alpha T_1} - 1}} \]

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\[ \theta = 0.03\% \]
\[ r(0) = -0.40\% \]
One Factor Hull-White Model
Mean Reversion Parameter – Example 4
Effect on Bermudan swaption price

- Sample Bermudan Swaption
  - Truly calibrated HW1F
  - N=100mln
  - yearly call dates
Mean Reversion Parameter
Example 5a: CMS10y-K at T=30y

\( \alpha = 5\% \)
Mean Reversion Parameter
Example 5b: $T=30y$ CMS10y-K

$\alpha = -5\%$
Mean Reversion Parameter
Example 5c: CMS10y at T=30y

\[ \alpha = -10\% \]
Mean Reversion Parameter

Example 6a: Simulation 6m EURIBOR cashflows

$\alpha = -1\%$
Mean Reversion Parameter
Example 6a: Simulation 6m EURIBOR cashflows

\[ \alpha = -10\% \]
Mean Reversion Parameter
Negative mean reversion (MR)

- Model volatility $\sigma(t)$
  - Decreases in $t$ for $a<0$
  - In general not all swaption prices can be matched perfectly
  - There exists a MR-dependent maximum maturity until which perfect calibration to European swaptions is possible

<table>
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<th>reversion ($\kappa$)</th>
<th>$t_{\text{max}}$</th>
<th>reversion ($\kappa$)</th>
<th>$t_{\text{max}}$</th>
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<td>10</td>
<td>-0.1</td>
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</table>
Mean Reversion Parameter
Optimal Mean Reversion 1

- Find MR leading to the closest match to the counterparty prices
- Market implied mean reversion
- Different optimality criteria on the portfolio level are possible
- Different optimization level are possible
  - deal level
  - CP level
  - global

![Graph showing price as a function of mean reversion](image)

$a^* = 3.5\%$  

**CP price**
Mean reversion optimized at counterparty level varies between -2% and -6%

(20 trades, 10 counterparts, three different optimality criteria)
## Mean Reversion Parameter
### Optimal Mean Reversion 3

<table>
<thead>
<tr>
<th>Optimal MR</th>
<th>Netted MtM Diff, EUR</th>
<th>Median Diff to N, bp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summit logic (&gt;2%)</td>
<td>9.677.732</td>
<td>116</td>
</tr>
<tr>
<td>Deal level (avr -3.2%)</td>
<td>2.485.357</td>
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<tr>
<td>CP level - sqrt (avr -3.3%)</td>
<td>2.699.485</td>
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<tr>
<td>Global – sqrt (-3.1%)</td>
<td>2.374.666</td>
<td>20</td>
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Effect of the mean reversion on MtM differences with counterparties
(20 trades, 10 counterparts)
Final remarks

- Mean reversion parameter controls the price level of Bermudans
- Market implies negative MR values in HW1F framework
  - Not intuitive
  - Theoretically hard to justify for limit cases
  - HW1F model cannot be perfectly calibrated anymore
  - HW1F model reaches its applicability limits

- **It is still working!**
  - Regular monitoring and update of the mean reversion is necessary

- Future work in ORE
  - Swaptions with amortizing notional, rate or spread

Thank you very much for your attention