ORE in Pricing of Bermudan Swaptions:
Client Experience from Model Validation

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Main points

- pbb and Quaternion joint venture on pricing of Bermudan swaptions
- Hull-White 1F (LGM)
- In-depth performance analysis

Agenda

1. Background

2. Hull-White Model

3. Mean Reversion Parameter

4. Final remarks

Background
Bermudan swaption project

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Start date	October 07th, 2016
End date	October 07th, 2030 (if not called before)
Party B pays to Party A	1,63%
	Act./Act. ICMA, following, unadjusted, roll date: October 07th
Party A pays to Party B	October 07th 2016 – October 07th, 2019: 3-months- Euribor + 70 bps,
	October 07th 2019 – October 07th, 2030: 3-months- Euribor + 100 bps,
	Act./360 adj. mod. foll., roll dates: 07th of the months January, April, July, October
Trade Date	September 30th, 2016
Call right for Party B	Party B has the right to call the swap on October 07th, 2019, 2022, 2025 and 2028

- ✓ Swap until final maturity
- ✓ Option to cancel swap => Bermudan Swaption
- ✓ Price is sensitive to the intertemporal correlation

- ✓ Reach portfolio of Bermudan callable swaps
- ✓ Daily prices from major investment banks in collateral management
- ✓ Analysis of model and market prices is possible

One Factor Hull-White Model
Definition

- Short rate process SDE

$$dr(t) = (\theta(t) - ar(t))dt + \sigma(t)dW(t)$$

$\theta(t)$ - a function may be calculated from the discount factors

$W(t)$ - standard Brownian motion

$\sigma(t)$ - piecewise constant model volatility (vector)

a - mean reversion parameter (scalar)

- Short rate *r(t)* is normally distributed

$$r(t) \sim \mathcal{N}\left(e^{-\alpha t} r(0) + \frac{\theta}{\alpha}\left(1 - e^{-\alpha t}\right), \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha t}\right)\right)$$

One Factor Hull-White Model
Calibration of model parameters

$$dr(t) = (\theta(t) - ar(t))dt + \sigma(t)dW(t)$$

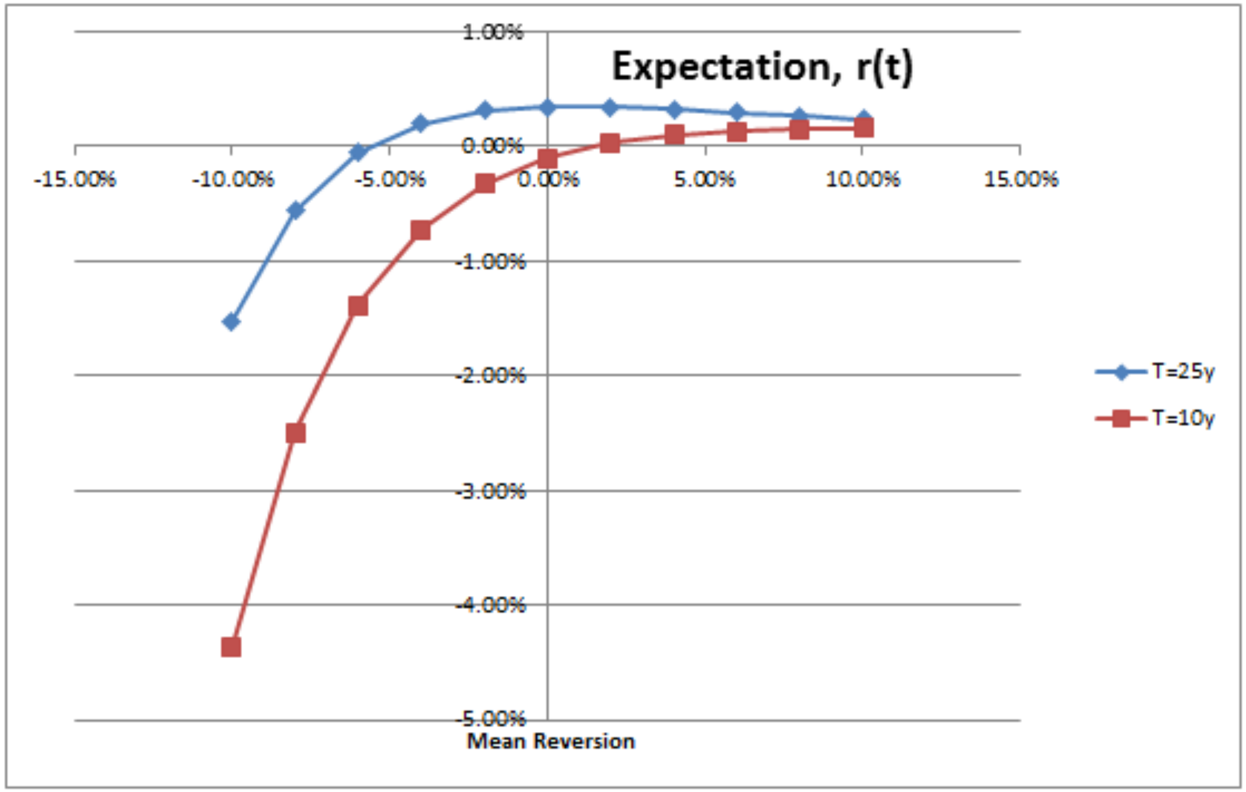
- Model volatility $\sigma(t)$
  - ✓ Calibrated on co-terminal European swaptions for given strikes
  - ✓ Has as many “steps” as calibrating swaptions
  - ✓ Iteratively stripped to match prices of all swaptions
- Mean Reversion a
  - ✓ Controls intertemporal correlation
  - ✓ “Historically estimated”
  - ✓ “Implied to produce certain volatility shape”
  - ✓ “Somehow set”
  - ✓ Ultimately freely selectable, or ?

Mean Reversion Parameter
Example 1: Expectation

$$r(t) \sim \mathcal{N}\left(e^{-\alpha t} r(0) + \frac{\theta}{\alpha}\left(1 - e^{-\alpha t}\right), \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha t}\right)\right)$$

$$\mathrm{corr}(r(T_1), r(T_2)) = \sqrt{\frac{e^{2aT_2} - 1}{e^{2aT_1} - 1}}$$

$$\sigma = 0.65\%$$
$$\theta = 0.03\%$$
$$r(0) = -0.40\%$$



Mean Reversion Parameter
Example 2: Variance

$$r(t) \sim \mathcal{N}\left(e^{-\alpha t} r(0) + \frac{\theta}{\alpha}\left(1 - e^{-\alpha t}\right), \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha t}\right)\right)$$

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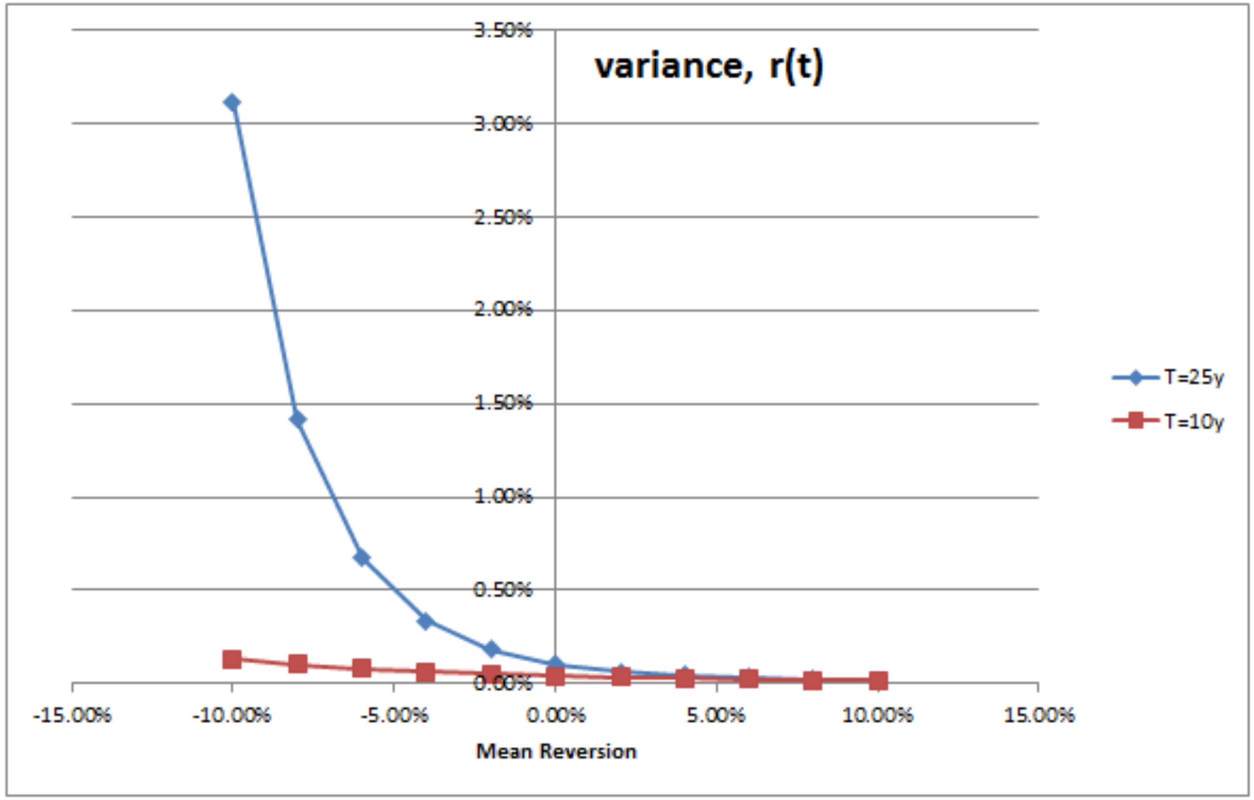

Mean Reversion Parameter
Example 3: Correlation

$$r(t) \sim \mathcal{N}\left(e^{-\alpha t} r(0) + \frac{\theta}{\alpha}\left(1 - e^{-\alpha t}\right), \frac{\sigma^2}{2\alpha}\left(1 - e^{-2\alpha t}\right)\right)$$

$$\operatorname{corr}(r(T_1), r(T_2)) = \sqrt{\frac{e^{2aT_2} - 1}{e^{2aT_1} - 1}}$$

$$\sigma = 0.65\%$$
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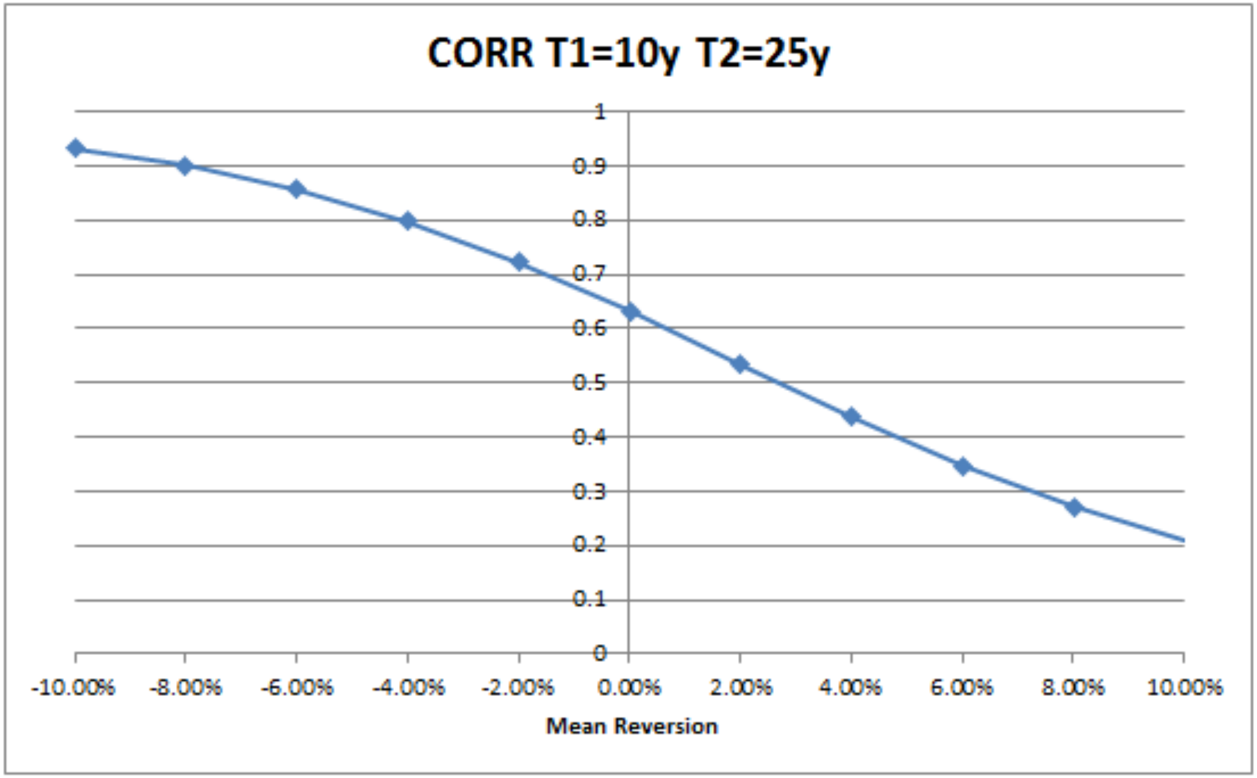
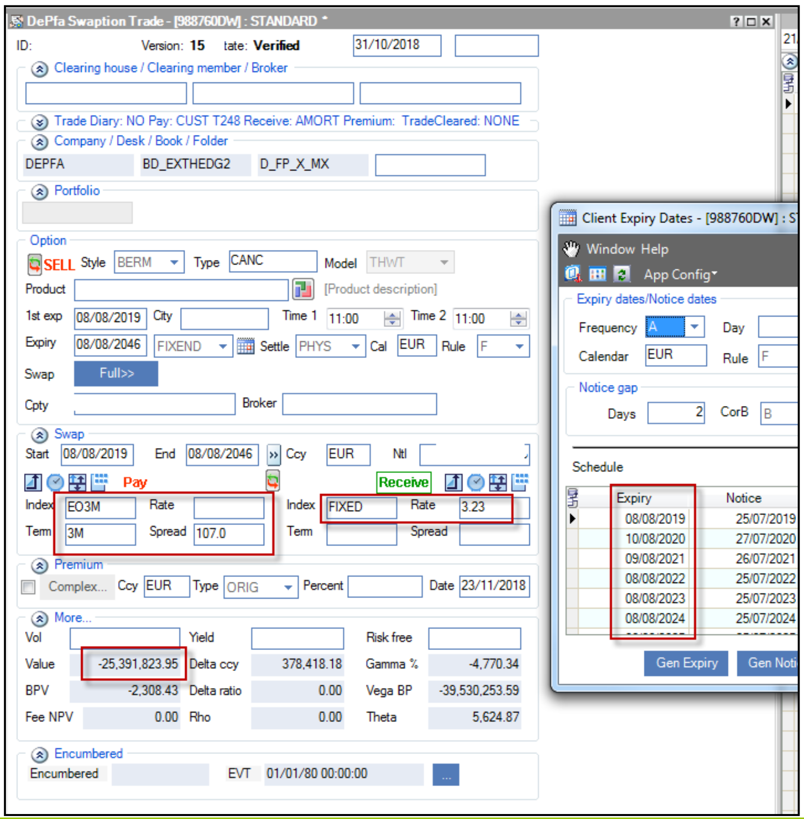
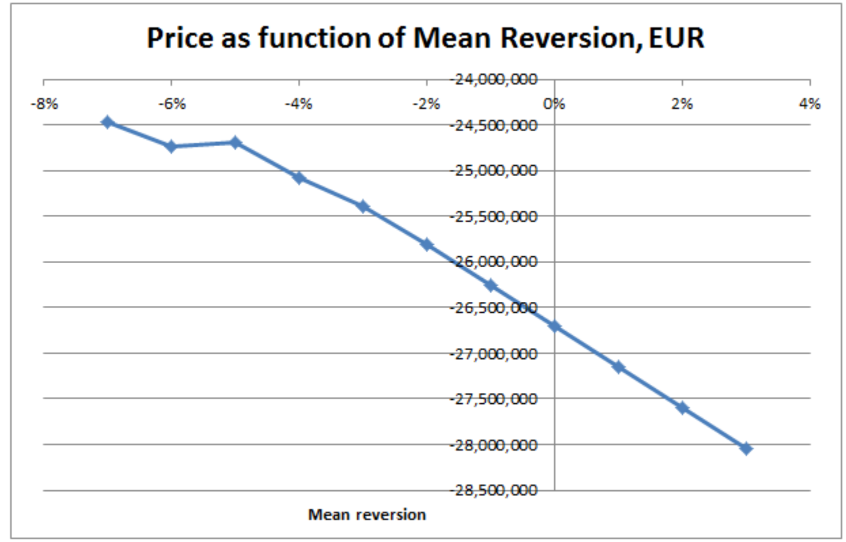
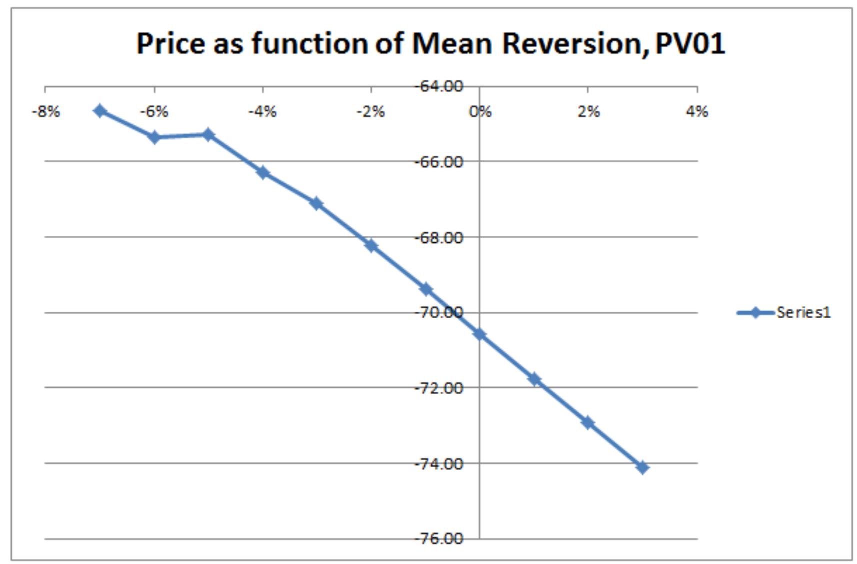


One Factor Hull-White Model
Mean Reversion Parameter – Example 4
Effect on Bermudan swaption price

- Sample Bermudan Swaption
  - Truly calibrated HW1F
  - N=100mln
  - yearly call dates

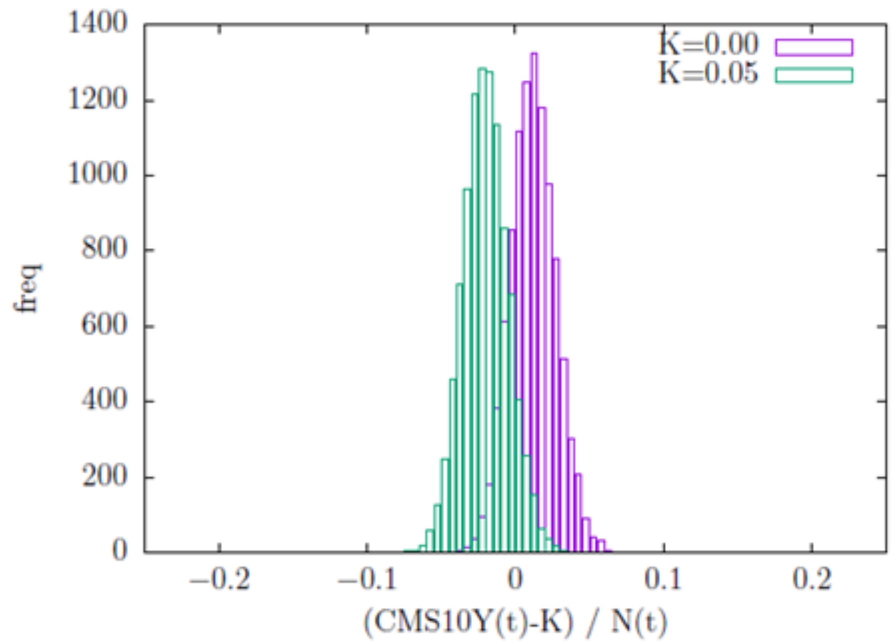






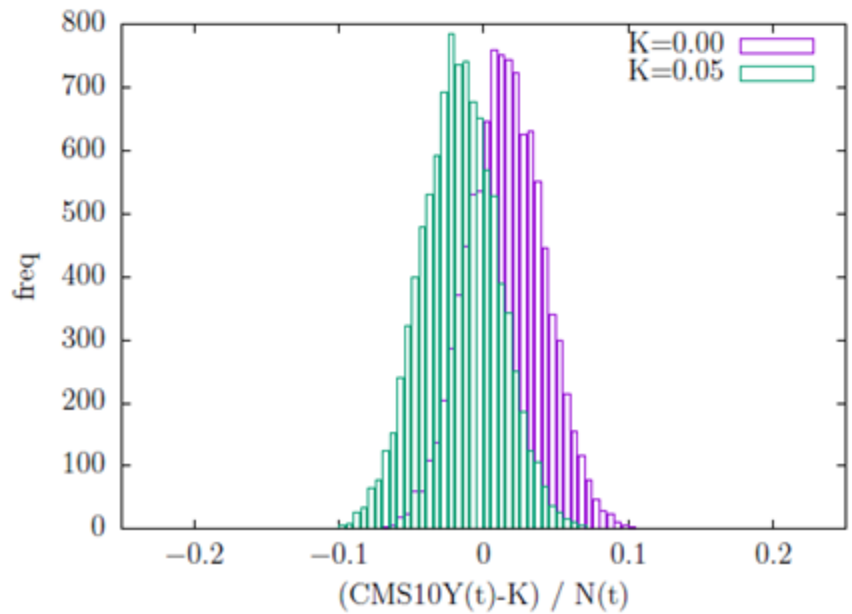
Mean Reversion Parameter
Example 5a: CMS10y-K at T=30y

$a = 5\%$



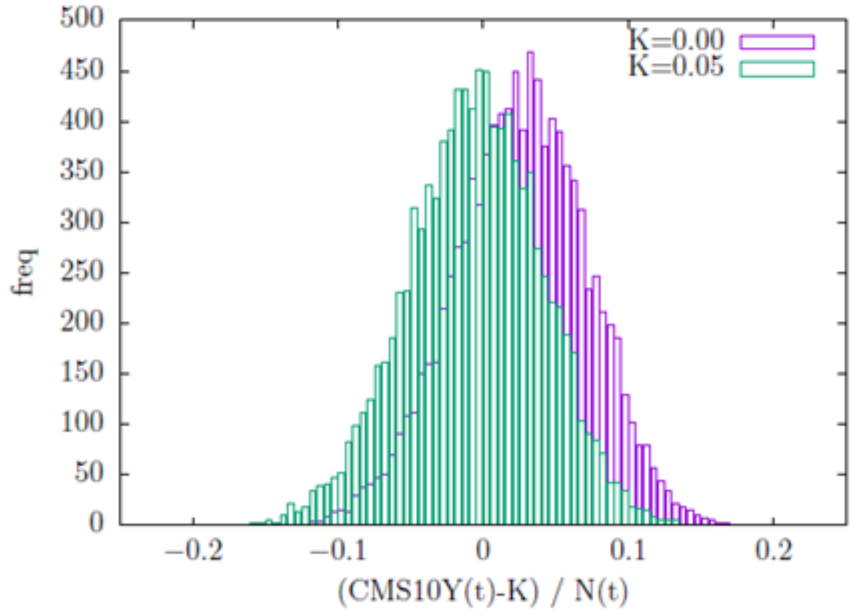
Mean Reversion Parameter
Example 5b: T=30y CMS10y-K

$a = -5\%$



Mean Reversion Parameter
Example 5c: CMS10y at T=30y

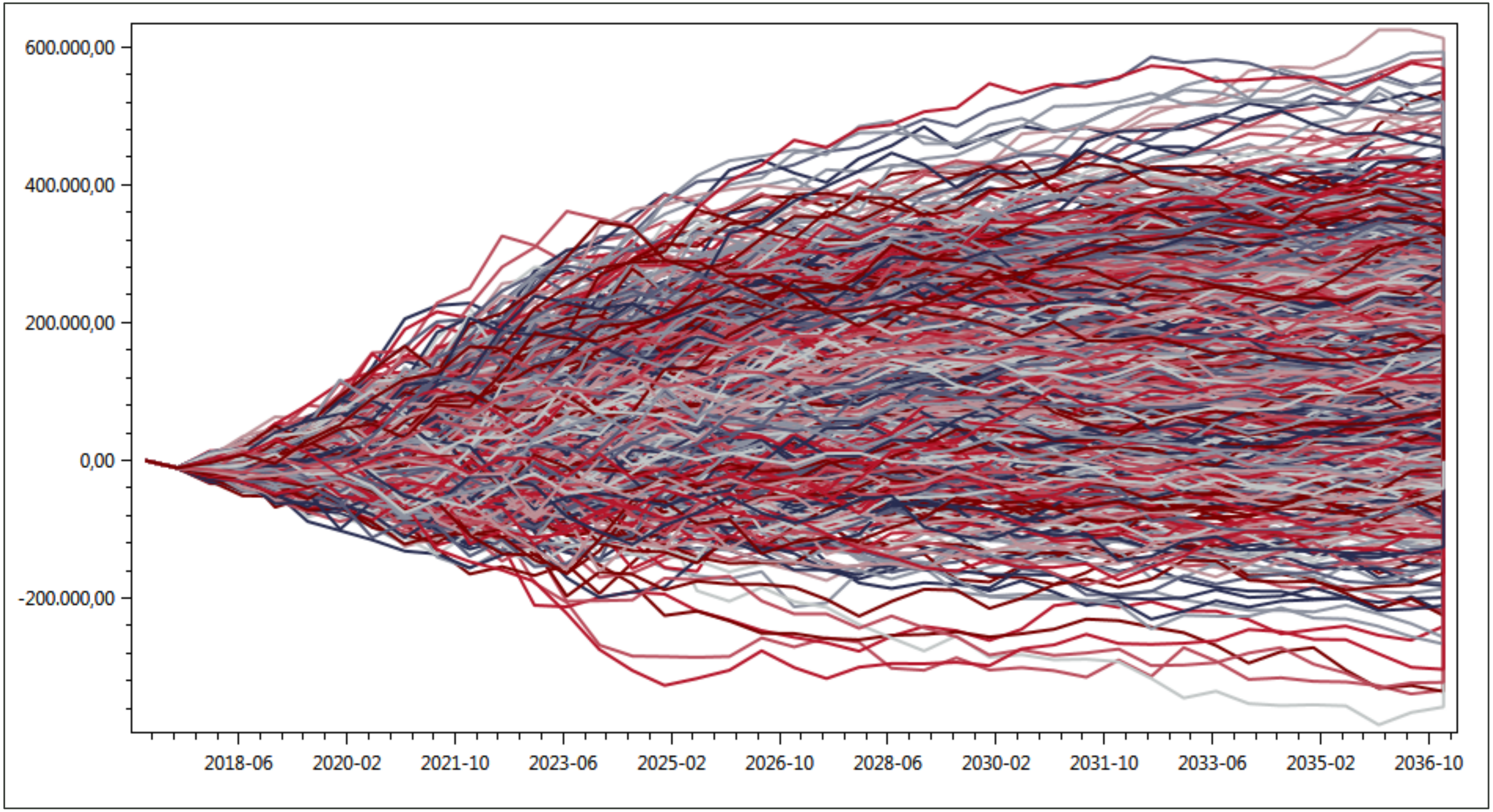
$a = -10\%$



Mean Reversion Parameter

Example 6a: Simulation 6m EURIBOR cashflows

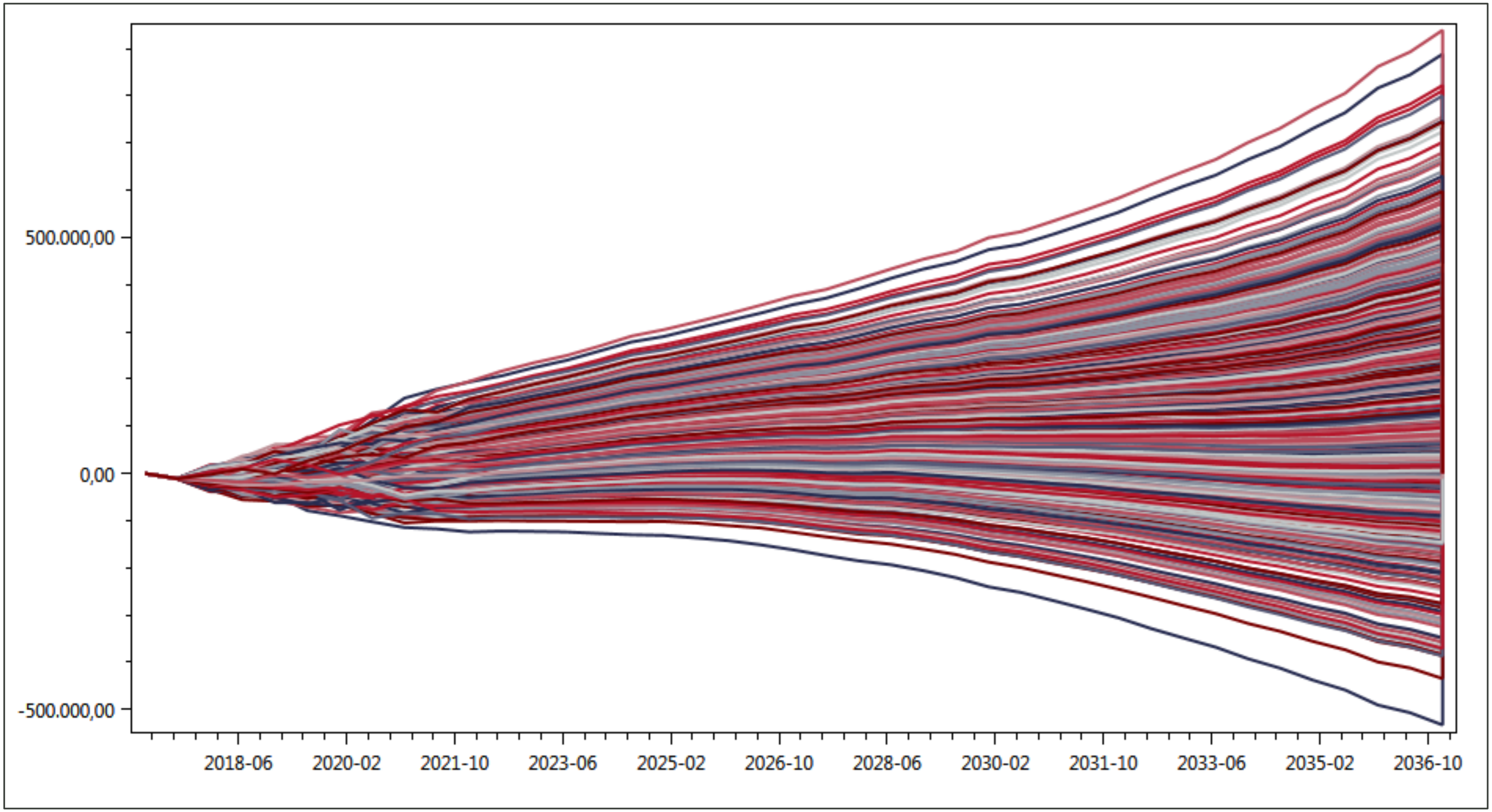
$a = -1\%$



Mean Reversion Parameter

Example 6a: Simulation 6m EURIBOR cashflows

$a = -10\%$



Mean Reversion Parameter
Negative mean reversion (MR)

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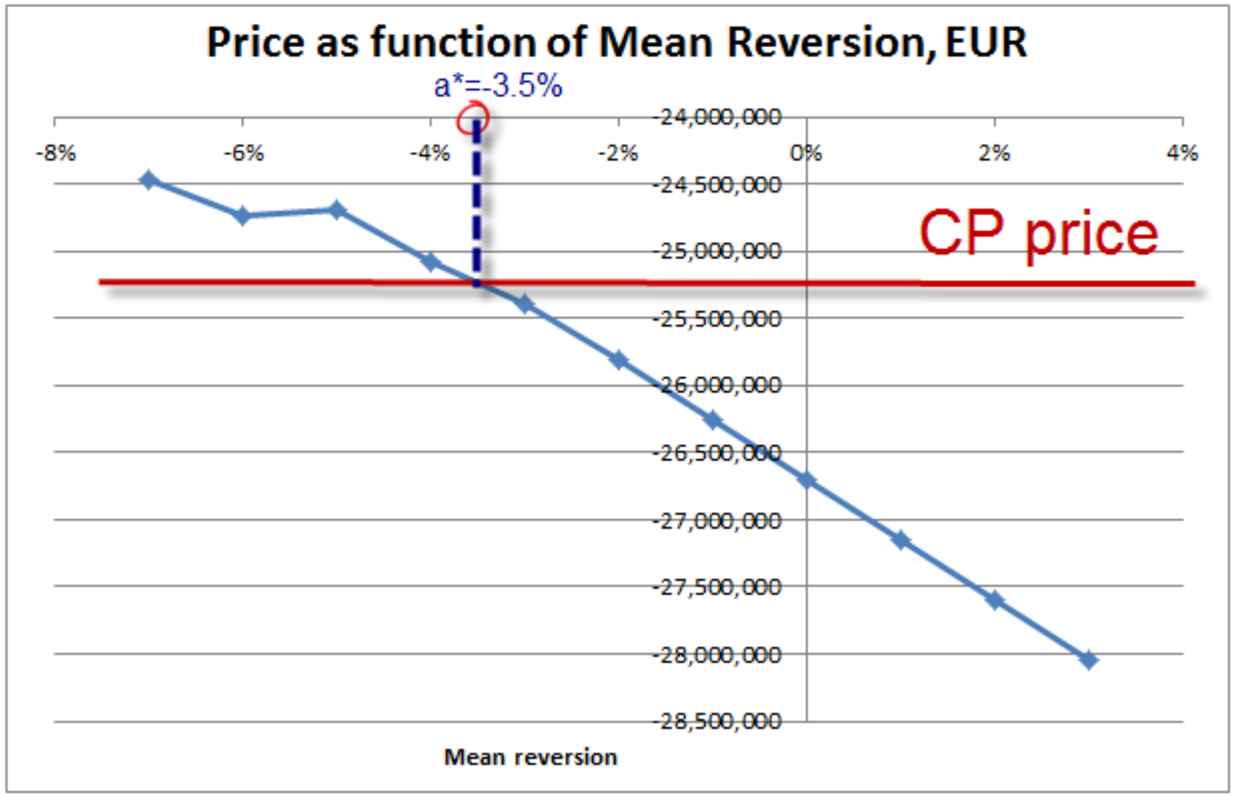
- Model volatility $\sigma(t)$
  - ✓ Decreases in *t* for *a<0*
  - ✓ In general not all swaption prices can be matched perfectly
  - ✓ There exists a MR-dependent maximum maturity until which perfect calibration to European swaptions is possible

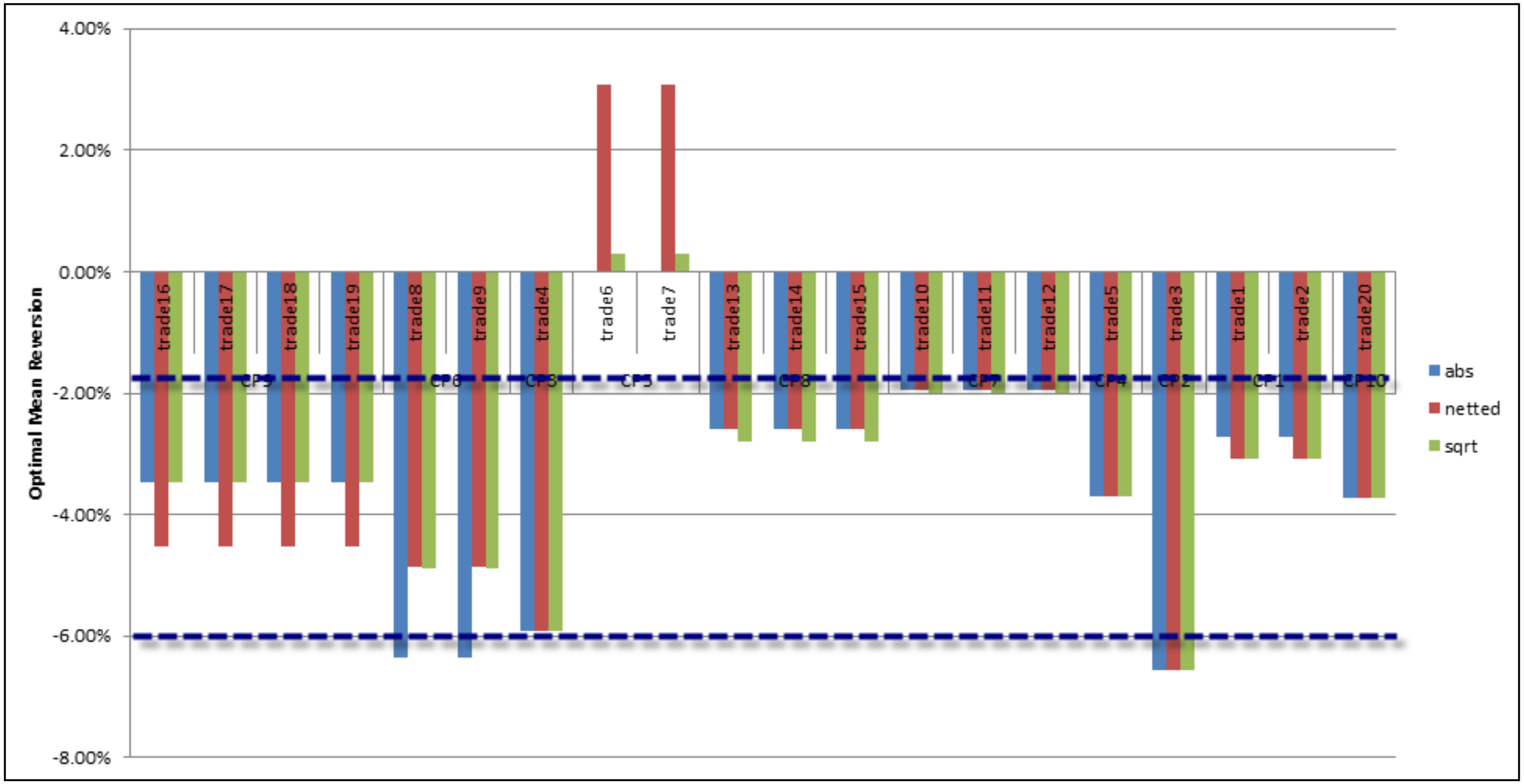
reversion (κ)	$t_{\max}$	reversion (κ)	$t_{\max}$
-0.0025	200	-0.0525	10
-0.005	100	-0.055	9
-0.0075	67	-0.0575	9
-0.01	50	-0.06	8
-0.0125	40	-0.0625	8
-0.015	33	-0.065	8
-0.0175	29	-0.0675	7
-0.02	25	-0.07	7
-0.0225	22	-0.0725	7
-0.025	20	-0.075	7
-0.0275	18	-0.0775	6
-0.03	17	-0.08	6
-0.0325	15	-0.0825	6
-0.035	14	-0.085	6
-0.0375	13	-0.0875	6
-0.04	13	-0.09	6
-0.0425	12	-0.0925	5
-0.045	11	-0.095	5
-0.0475	11	-0.0975	5
-0.05	10	-0.1	5

Mean Reversion Parameter
Optimal Mean Reversion 1

- Find MR leading to the closest match to the counterparty prices
- Market implied mean reversion
- Different optimality criteria on the portfolio level are possible
- Different optimization level are possible
  - deal level
  - CP level
  - global



Mean Reversion Parameter
Optimal Mean Reversion 2



Mean reversion optimized at counterparty level varies between -2% and -6%

(20 trades, 10 counterparts, three different optimality criteria)

Mean Reversion Parameter
Optimal Mean Reversion 3

Optimal MR	Netted MtM Diff, EUR	Median Diff to N, bp
Summit logic (>2%)	9.677.732	116
Deal level (avr -3.2%)	2.485.357	8
CP level - sqrt (avr -3.3%)	2.699.485	5
Global – sqrt (-3.1%)	2.374.666	20

Effect of the mean reversion on MtM differences with counterparties
(20 trades, 10 counterparts)

Final remarks

- Mean reversion parameter controls the price level of Bermudans
- Market implies negative MR values in HW1F framework
  - Not intuitive
  - Theoretically hard to justify for limit cases
  - HW1F model cannot be perfectly calibrated anymore
  - HW1F model reaches its applicability limits

- **It is still working!**
- Regular monitoring and update of the mean reversion is necessary

- Future work in ORE
  - Swaptions with amortizing notional, rate or spread

<u>Thank you very much for your attention</u>