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Model Validation of xVA using ORE

ORE User Meeting

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Overview

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The Open Source Risk Engine's objective is to provide a free/open source platform for risk analytics and xVA. It is based on QuantLib and grew from work developed by market professionals and academics.
The aim is to build on QuantLib's simulation models, instruments, and engines, and work by professionals and academics. It brings large-scale open-source risk models to the public domain to facilitate research and understanding through simple interfaces for trade/market data and system configurations, and simple launchers, see *opensourcerisk.org*.

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Overview

1. Hybrid multi-asset IR/FX model
   - Validation of Risk Factor Models and Calibration
   - Exposure Benchmarking using ORE
   - Regression Test based on Hull-White Model
   - Collateralization
2. Prototype of a Cheyette Model
3. Appendix
   - Markovian Representation
   - European swaption pricing in TS

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Validation Tasks using ORE

- Exposure simulation and xVA calculation
- Method based on Least Squares Monte-Carlo (LSM) simulation
- IR and FX models and products were considered
- ORE was applied for:
  - Modeling assumptions of the risk factors (RF)
  - Calibration methodology of RF models
  - Analysis / Benchmarking of LSM - in particularly the regression error
  - Benchmarking exposure profiles for collateralized / un-collateralized netting sets
- The validation scope:
  - IR/FX RF models and products.

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Risk Factor (RF) models

IR Model: Hull-White model + time-dependent *reversion* and *volatility*:

$$dr(t) = \lambda(t)(\theta(t) - r(t))dt + \sigma(t)dW(t),$$

W Brownian motion under $\mathbb{Q}$, θ chosen such that the initial term structure is matched.

FX Model: Based on Hull-White modeling for the domestic r_d and the foreign rate r_f and GBM for the exchange rate X:

$$\begin{aligned} dr_d(t) &= \lambda_d(t)(\theta_d(t) - r_d(t)))dt + \sigma_d(t)dW_d(t), \\ dr_f(t) &= \lambda_f(t)(\theta_f(t) - r_f(t))dt - \rho_{f,X}\sigma_f(t)\sigma_X(t)X(t)dt + \sigma_f(t)dW_f(t) \\ dX(t) &= (r_d(t) - r_f(t))X(t)dt + \sigma_X(t)X(t)dW_X(t), \end{aligned}$$

W_d, W_f, W_X correlated Brownian motions under $\mathbb{Q}$ (domestic), θ_d and θ_f are chosen such that the term structures are matched.

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Validation of RF Model and its Calibration in ORE

- IR model in ORE: 1F-HW model usign the LGM formulation (being equivalent to HW modeling assumption).
- FX model in ORE: Garman-Kohlhagen model, that is GBM for the exachange rate, with stochastic drift.
- In ORE both under Linear Gauss Markov (LGM) ansatz and. See, Lichters *et. al.* [2].
- With the help of ORE's xml interface the calibration strategy of the IR model is replicated and the fit of the model on the non-calibration instruments are analyezed (cross validation)
- The negative mean reversion phenomenon of the model is analyzed.

Cross Validation of Non-Calibration Instruments

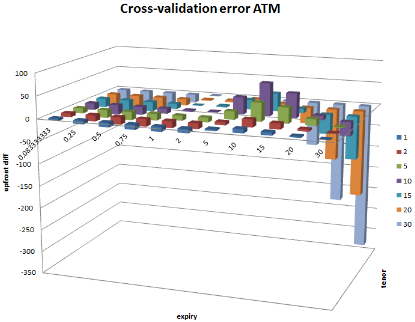

Figure: Upfront differences for ATM swaptions not necessary calibrated to.

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Cross Validation of Non-Calibration Instruments

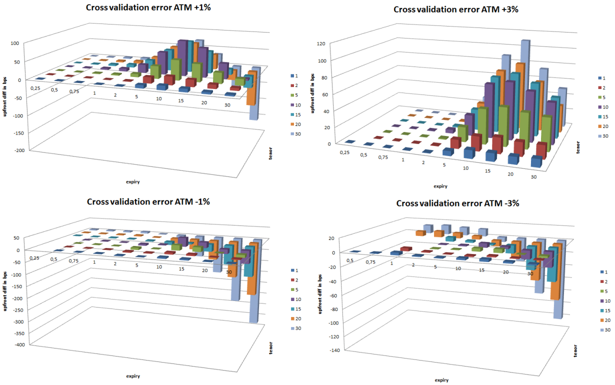

Figure: Upfront differences for ATM + 1% (top left), ATM + 3% (top left), ATM - 1% (bottom left) and ATM - 3% (bottom right) swaptions

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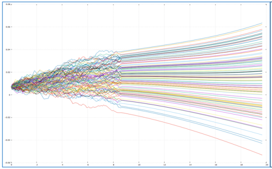
Negative Mean-Reversion in 1F-HW model

Negative mean reversion changes the basic modelling assumption of mean reversion. It turns it into a mean repelling model:



reversion (κ)	$t_{\max}$	reversion (κ)	$t_{\max}$
-0.0025	200	-0.0525	10
-0.005	100	-0.055	9
-0.0075	67	-0.0575	9
-0.01	50	-0.06	8
-0.0125	40	-0.0625	8
-0.015	33	-0.065	8
-0.0175	29	-0.0675	7
-0.02	25	-0.07	7
-0.0225	22	-0.0725	7
-0.025	20	-0.075	7
-0.0275	18	-0.0775	6
-0.03	17	-0.08	6
-0.0325	15	-0.0825	6
-0.035	14	-0.085	6
-0.0375	13	-0.0875	6
-0.04	13	-0.09	6
-0.0425	12	-0.0925	5
-0.045	11	-0.095	5
-0.0475	11	-0.0975	5
-0.05	10	-0.1	5

Figure: Effect of setting the mean reversion to negative value.

The paths are now drifting away from the long term mean. In this case the *long term mean* looses its characteristics.

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Exposure Comparative Tests ORE vs the validated system

Test Steps

1. Calculate the exposure profile and extract trade data and market data from the system's proprietary XML
2. Convert trade data, market data and model parameters from the system XML to ORE XML
3. Calculate the exposure profile in ORE, plot the exposure profiles from the system and ORE

XML Input of ORE Trade

```xml
<Portfolio>
  <Trade id="FXFWD_EURUSD_1Y">
    <TradeType>FxForward</TradeType>
    <Envelope>
      <CounterParty>CPTY_A</CounterParty>
      <NettingSetId>CPTY_A</NettingSetId>
      <AdditionalFields/>
    </Envelope>
    <FxForwardData>
      <ValueDate>2019-03-28</ValueDate>
      <BoughtCurrency>EUR</BoughtCurrency>
      <BoughtAmount>1000000</BoughtAmount>
      <SoldCurrency>USD</SoldCurrency>
      <SoldAmount>1272900.4663024701</SoldAmount>
    </FxForwardData>
  </Trade>
</Portfolio>
```

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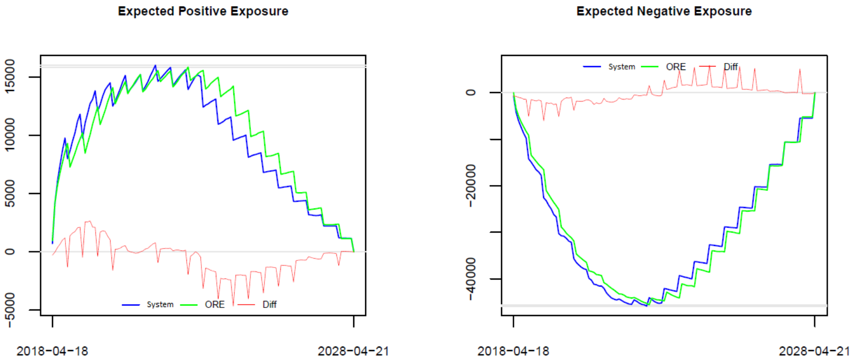
10 Year At-The-Money IR Swap



- The left image shows the expected positive exposures of the system (blue), the expected positive exposures of ORE (green) and their absolute difference (red)
- The right image shows the expected negative exposures of the system (blue), the expected negative exposures of ORE (green) and their absolute difference (red)

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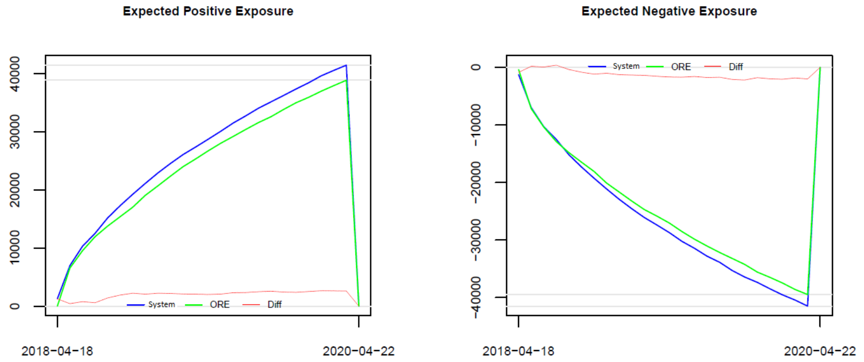
2 Year EUR/USD FX Forward



- The left image shows the expected positive exposures of the system (blue), the expected positive exposures of ORE (green) and their absolute difference (red)
- The right image shows the expected negative exposures of the system (blue), the expected negative exposures of ORE (green) and their absolute difference (red)

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Approximation of Historical PV Distribution via Least-Square Method (LSM)

Test Part I: Regression test based on historical data

- The first part of the test was carried out using ORE
- Test samples consists of the historical PVs and the historical market data
- The historical 5-year discount factor was chosen as the risk factor for the test instruments
- The regression was between historical PVs and corresponding log-changes of the RF
- The regression PVs were then obtained by the regression parameters and the log-changes of the RF

Test Part II: Regression test regarding HW model

- For the second part we replaced the historical discount curves with the discount curves generated by the 1 factor HW model
- The 1F HW model generates a distribution of discount curves by adjusting the random numbers
- Select one of them such that it optimally fits the corresponding historical discount curve
- The regression was between the historical PVs and corresponding log changes of the 5-year HW discount factor

Observation - Regression Test

- The regression based on the 1F-HW model can not sufficiently well approximate the distribution of historical PVs.
- The 1F-HW model can not reproduce the historical discount curves sufficiently well.

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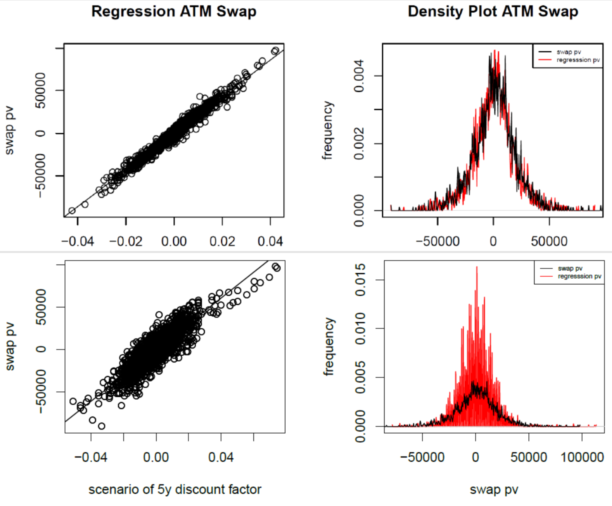
4 Year At-The-Money Swap Forward Starting in 1 Year



Regression and Density Comparison based on Historical Data

Regression and Density Comparison regarding 1F HW Model

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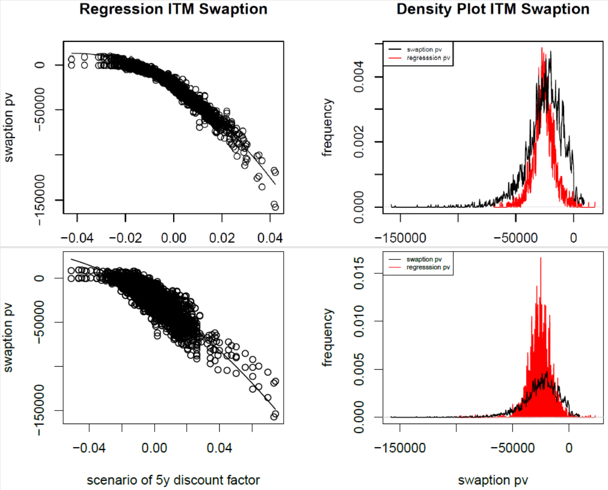
In-The-Money (1 Year × 4 Year) - IR Swaption



Regression and Density Comparison based on Historical Data

Regression and Density Comparison regarding Hull-White Model

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Collateralization

- The next task was to benchmark the expected collateral and collateralized exposure.
- Only the variation margin was in scope (no initial margin).
- Implementing a collateral model from scratch is very time consuming and really nitty gritty stuff.
- By using ORE's collateral model we compared exposure profiles.

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Trolle-Schwartz Model

A model currently in prototyping state in ORE is an interest rate model applied for the exposure calculation.

- IR model: Trolle-Schwartz Model [1].
- HJM type model with (unspanned) stochastic volatility being a square root process.
- Advantages :
  - Markovian representations for instantenous forward rates and zero bond prices
  - Swap / Libor rates dynamics available via Ito-formula

  $\Rightarrow$ Efficent simulation of future market scenarios using Monte Carlo simulation

  - Semi-closed form solutions for European swaptions

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Trolle-Schwartz Mode in ORE+

Work in progress in ORE+:

- TS model and the corresponding stochastic process is implemented.
- Analytic and Monte-Carlo pricing engines for European Swaptions are implemented.
- Calibration tests have been conducted
- Exposures profiles - IR swaps up to now are available -

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Trolle-Schwartz Model

State Variables

$$dx(t) = -\gamma x(t)\delta t + \sqrt{v(t)}dW^{\mathbb{Q}}(t)$$
$$dv(t) = \kappa(\theta - v(t))\delta t + \sigma\sqrt{v(t)}\left(\rho dW^{\mathbb{Q}}(t) + \sqrt{1-\rho^2}dZ^{\mathbb{Q}}(t)\right),$$

and six additional equations

$$d\phi_1(t) = (x(t) - \gamma\phi_1(t))dt$$
$$d\phi_2(t) = (v(t) - \gamma\phi_2(t))dt$$
$$d\phi_3(t) = (v(t) - 2\gamma\phi_3(t))dt$$
$$d\phi_4(t) = (\phi_2 - \gamma\phi_4(t))dt$$
$$d\phi_5(t) = (\phi_3 - 2\gamma\phi_5(t))dt$$
$$d\phi_6(t) = (2\phi_5 - 2\gamma\phi_6(t))dt.$$

For the model there exist a Markovian Representation for (instantaneous) Forward Rates and Zero Bonds.

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European Call Prices

- For the calibration procedure the fast and accurate pricing of European Swaptions is crucial.
- The original paper [1] gives an approximation formula based on the pricing of options on Zero Coupon Bonds (see Duffie *et. al* [3]) and one applying *Stochastic Duration* (see [4]).
- In ORE+ we implemented the semi-analytical swaption pricing using the *characteristic function*.
- In order to backtest the implementation and applying it for benchmarking a Monte Carlo pricing engine is implemented as well.

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Analytic versus MC pricing for ATM European Swaptions

expiry	terms	strike	Analytic-Price	MC-Price	MC error bounds	upfont diff
1M	1Y	0	0.000538407	0.00053677	[0.0005343141;0.0005392256]	0.0163692
1M	5Y	0	0.003043611	0.003042988	[0.003029014;0.003056963]	0.0062286
1M	10Y	0	0.00638258	0.006379336	[0.006349882;0.006408789]	0.0324422
1M	20Y	0	0.01197659	0.01200982	[0.01195381;0.01206582]	0.3322411
1Y	1Y	0	0.001901946	0.001903611	[0.00189516;0.001912063]	0.0166563
1Y	5Y	0	0.01052716	0.0105489	[0.01050149;0.01059632]	0.2173906
1Y	10Y	0	0.02178112	0.0218268	[0.02172697;0.02192662]	0.4567794
1Y	20Y	0	0.04044624	0.04056714	[0.0403755;0.04075878]	1.208999
5Y	1Y	0	0.004466735	0.004442504	[0.004422836;0.004462173]	0.2423085
5Y	5Y	0	0.0232338	0.02312228	[0.02301703;0.02322754]	1.115125
5Y	10Y	0	0.04588883	0.04571886	[0.04550287;0.04593486]	1.699672
5Y	20Y	0	0.08129297	0.08109157	[0.08068205;0.08150109]	2.014026
10Y	1Y	0	0.006068563	0.006092204	[0.006063941;0.006120467]	0.2364074
10Y	5Y	0	0.03015976	0.03025692	[0.03011106;0.03040279]	0.9716501
10Y	10Y	0	0.05749326	0.05776784	[0.05747538;0.05806029]	2.74582
10Y	20Y	0	0.09794462	0.0985932	[0.09804996;0.09913645]	6.485833
20Y	1Y	0	0.006701042	0.006729522	[0.00669198;0.006767065]	0.2848017
20Y	5Y	0	0.03186418	0.03203104	[0.03184372;0.03221835]	1.668528
20Y	10Y	0	0.05861921	0.05907998	[0.05871421;0.05944576]	4.6077
20Y	20Y	0	0.09574228	0.09689308	[0.09623355;0.09755261]	11.50794

Figure: Analytic versus MC prices of European swaptions

Numerical Challenges in European (semi-) analytic Swaption Pricing

- Basic:
  - Root finding algorithm for the calculation of stochastic duration: QuantLib's Brent root finding algorithm is used.
  - Numerical solution of the Ricatti equations: QuantLib's Adaptive Runga-Kutta ODE solver is utilized.
- Tricky:
  - Numerical integration the inverse fourier transform: GSL's adaptive Gauss-Kronrod Integration with 61 points (QNG). Problems with the oscilating integrands may occur for some parameter settings. See the next slides. Work on progress using GSL's adaptive integration for oscillatory functions (QAWO).

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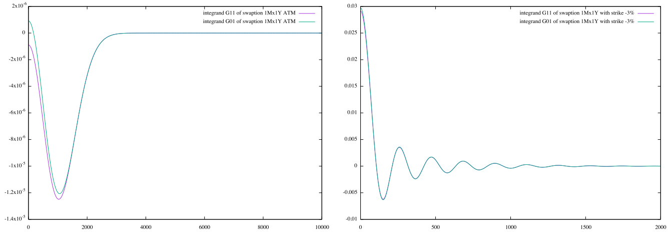

Figure: Integrands for the case of a 1Mx1Y Swaption for different strike values.

Exposure profile IR swap

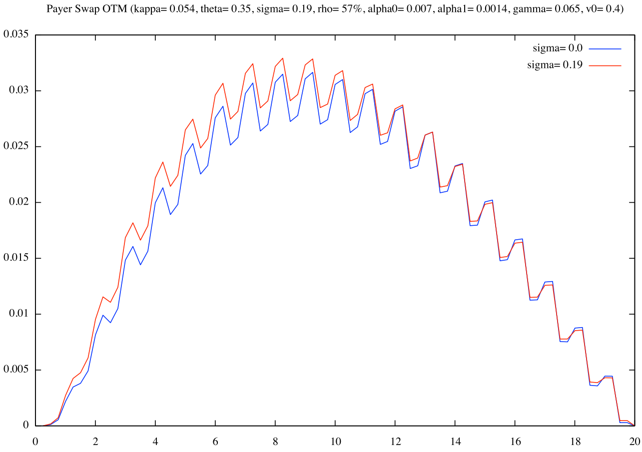

Figure: Example: Exposure profile of an IRS using different values for σ.

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Markov Representation of the (instantaneous) Forward Rate

$$f(t,T) = f(0,T) + B^f_x(T-t)x(t) + \sum_{j=1}^{6} B^f_{\phi_j}(T-t)\phi_j(t),$$

with

$$B^f_x(\tau) := (\alpha_0 + \alpha_1\tau)\, e^{-\gamma\tau} \qquad B^f_{\phi_1}(\tau) := \alpha_1 e^{-\gamma\tau}$$

$$B^f_{\phi_2}(\tau) := \left(\frac{\alpha_1}{\gamma}\right)\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right)(\alpha_0 + \alpha_1\tau)\, e^{-\gamma\tau}$$

$$B^f_{\phi_3}(\tau) := -\left(\frac{\alpha_0\alpha_1}{\gamma}\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right) + \frac{\alpha_1}{\gamma}\left(\frac{\alpha_1}{\gamma} + 2\alpha_0\right)\tau + \frac{\alpha_1^2}{\gamma}\tau^2\right) e^{-2\gamma\tau}$$

$$B^f_{\phi_4}(\tau) := \frac{\alpha_1^2}{\gamma}\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1}\right) e^{-\gamma\tau}$$

$$B^f_{\phi_5}(\tau) := -\frac{\alpha_1}{\gamma}\left(\frac{\alpha_1}{\gamma} + 2\alpha_0 + 2\alpha_1\tau\right) e^{-2\gamma\tau}, \qquad B^f_{\phi_6}(\tau) := -\frac{\alpha_1^2}{\gamma} e^{-2\gamma\tau}.$$

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Markov Representation of the Zero Bond

$$P(t,T) = \frac{P(0,T)}{P(0,t)} \exp\left(B_x^P(T-t)x(t) + \sum_{j=1}^{6} B_{\phi_j}^P(T-t)\phi_j(t) \right),$$

with

$$B_x^P(\tau) = \frac{1}{\gamma}\left(\left(\frac{\alpha_1}{\gamma} + \alpha_0 \right)(e^{-\gamma\tau} - 1) + \alpha_1 \tau e^{-\gamma\tau} \right), \quad B_{\phi_1}^P(\tau) = \frac{\alpha_1}{\gamma}(e^{-\gamma\tau} - 1),$$

$$B_{\phi_2}^P(\tau) = \left(\frac{\alpha_1}{\gamma}\right)^2 \left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1} \right) \left(\left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1} \right)(e^{-\gamma\tau} - 1) + \tau e^{-\gamma\tau} \right),$$

$$B_{\phi_3}^P(\tau) = -\frac{\alpha_1}{\gamma^2} \left(\left(\frac{\alpha_1}{2\gamma^2} + \frac{\alpha_0}{\gamma} + \frac{\alpha_0^2}{2\alpha_1} \right)(e^{-2\gamma\tau} - 1) + \left(\frac{\alpha_1}{\gamma} + \alpha_0 \right) \tau e^{-2\gamma\tau} + \frac{\alpha_1}{2} \tau^2 e^{-2\gamma\tau} \right)$$

$$B_{\phi_4}^P(\tau) = \left(\frac{\alpha_1}{\gamma}\right)^2 \left(\frac{1}{\gamma} + \frac{\alpha_0}{\alpha_1} \right)(e^{-\gamma\tau} - 1), \quad B_{\phi_6}^P(\tau) = -\frac{1}{2}\left(\frac{\alpha_1}{\gamma}\right)^2 (e^{-2\gamma\tau} - 1),$$

$$B_{\phi_5}^P(\tau) = -\frac{\alpha_1}{\gamma^2} \left(\left(\frac{\alpha_1}{\gamma} + \alpha_0 \right)(e^{-2\gamma\tau} - 1) + \alpha_1 \tau e^{-2\gamma\tau} \right).$$

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Options on zero bonds

Zero coupon bond option with expiry T_{exp}, term T_{term} and strike K is given by

$$OZB(w;t,T_{exp},T_{term},K) = w\,K\,G_{0,w}(w\log K) - w\,G_{1,w}(w\log K),$$

where $w = \pm 1 \equiv$ put/call and $G_{a,b}(y)$ is given by

$$G_{a,b}(y) = \frac{1}{2}\Psi(a,t,T_{exp},T_{term}) - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left[\Psi(a+iub,t,T_{exp},T_{term})e^{-iuy}\right]}{u}\,du,$$

with $\Psi(u,t,T_0,T_1)$ having the form

$$\Psi(u,t,T_0,T_1) = \exp\Big(M(T_0-t) + N(T_0-t)v(t) + u\log P(t,T_1) + (1-u)\log P(t,T_0)\Big),$$

where the functions M and N are the solutions of the Ricatti equations:

$$\begin{aligned}\frac{dM(\tau)}{\tau} &= N(\tau)\kappa\theta \\ \frac{dN(\tau)}{\tau} &= N(\tau)\Big[-\kappa + \sigma\rho\,[uB_x(T_1 - T_0 + \tau)] + (1-u)B_x(\tau)\Big] \\ &\quad + 0.5\,N(\tau)^2\sigma^2 + 0.5\,u(u-1)\,[B_x(T_1,T_0+\tau) - B_x(\tau)]^2\,,\end{aligned}$$

with initial values $M(0) = 0$ and $N(0) = 0$.

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Stochastic Duration

The idea of the stochastic duration approach is to approximate a European option on a coupon bond with a (scaled) European option on a zero-coupon bond with maturity equal to the stochastic duration of the coupon bond.

Munk's result: The instantaneous variance of the relative price change of a zero-bond is:

$$Var_{ZB}(t,T) = B_x^2(T-t) \tag{1}$$

and for a coupon bond with coupons c_i over the period $[T_{i-1}, T_i]$ is given by:

$$Var_{CB}(t) = \left(\sum_{i=1}^{N_{fxd}} w_i B_x(T_i - t)\right)^2 \tag{2}$$

where $w_i = c_i P(t,Ti) / \sum_{i=1}^{N_{fxd}} c_i P(t,T_i)$.

$$D(t): \quad Var_{ZB}(t, t + D(t)) = Var_{CB}(t)$$

which is equivalent to solving the following root finding problem:

$$B_x^2(D(t)) = \left(\sum_{i=1}^{N_{fixed}} w_i B_x(T_i - t)\right)^2 \tag{3}$$

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Option on Coupon Bonds

Therefore the time t-price of a coupon bond option with expiry T_{exp}, term T_{term} (maturity of the underlying coupon bond) and strike K (denoted by $OCB(t, T_{exp}, T_{term}, K)$) is given by

$$OCB(w; t, T_{exp}, T_{term}, K) = \xi \, OZB(w; t, T_{exp}, t + D(t), \xi^{-1}) \qquad (4)$$

where

$$\xi = \frac{\sum_{i=1}^{N_{fxd}} c_i P(t, T_i)}{P(t, t + D(t))}.$$

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Pricing of swaptions with coupon-bond options

A payer (receiver) swaption is a call option on a swap rate but can also can be valued as a European put (call) option on a coupon bond and hence can be priced by using:

$$SO(w; t_0, T_{exp}, T_{term}) = E^Q\left[w\left(1 - \left[\sum_{l=1}^{N_{fxd}} c_l P^{OIS}(t_0, T_l) + \sum_{k=1}^{N_{flt}} \widetilde{c}_k P^{OIS}(t_0, T_k)\right]\right)^+\right]$$

where $w = \pm 1$ is the payer swaption and receiver swaption respectively and

$$c_l = \begin{cases} K\tau_{fxd}(T_{l-1}, T_l) & , if\ l = 1, \ldots, N_{fxd} - 1 \\ 1 + K\tau_{fxd}(T_{l-1}, T_l) & , if\ l = N_{fxd} \end{cases} \quad , \quad \widetilde{c}_k = -s(T_{k-1}, T_k)\tau_{flt}(T_{k-1}, T_k) \tag{5}$$

Then,

$$SO(w; t_0, T_{exp}, T_{term}) = \xi\, OZB(w; t, T_{exp}, t + D(t), \xi^{-1})$$

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Pricing of swaptions with coupon-bond options

where $D(t)$ is defined by

$$B_x^2(D(t)) = \left(\sum_{i=1}^{N_{fxd}} w_i B_x(T_i - t) + \sum_{j=1}^{N_{flt}} \widetilde{w}_j B_x(T_j - t) \right)^2$$

with

$$w_i = \frac{c_i P(t, Ti)}{\sum_{i=1}^{N_{fxd}} c_i P(t, T_i)}, \quad \widetilde{w}_j = \frac{\widetilde{c}_j P(t, Tj)}{\sum_{j=1}^{N_{flt}} \widetilde{c}_j P(t, T_j)}$$

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[1] A. B. Trolle and E. S. Schwartz, *A General Stochastic Volatility Model for the Pricing of Interest Rate Derivatives*, The Review of Financial Studies **22**

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